

$$\mathcal{L}[f] = \mathcal{L}[(u_0(n) - u_{\pi/2}(n)) \sin n + u_{\pi/2}(n) (\sin n + \cos n)]$$

$$= \mathcal{L}[\sin n] + \mathcal{L}[u_{\pi/2}(n) \cos n]$$

$$= \frac{1}{1+s^2} - \mathcal{L}[u_{\pi/2}(n) \sin(n - \pi/2)] \quad (\text{نمره ۳})$$

$$= \frac{1}{1+s^2} - \frac{e^{-\frac{\pi}{2}s}}{1+s^2} = \frac{1 - e^{-\frac{\pi}{2}s}}{1+s^2} \quad (\text{نمره ۲}) \textcircled{1}$$

$$\mathcal{L}[y'(n) + 2y(n) + \int_0^n y(t) dt] = (s+2)\mathcal{L}[y] + \mathcal{L}[1 * y] \quad (\text{نمره ۳})$$

$$= (s+2 + \frac{1}{s}) \mathcal{L}[y] = \left(\frac{s^2 + 2s + 1}{s} \right) \mathcal{L}[y] = \frac{(s+1)^2}{s} \mathcal{L}[y] \quad (\text{نمره ۳}) \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \Rightarrow \mathcal{L}[y] \frac{(s+1)^2}{s} = \frac{1 - e^{-\frac{\pi}{2}s}}{1+s^2}$$

$$\Rightarrow \mathcal{L}[y] = \frac{s(1 - e^{-\frac{\pi}{2}s})}{(1+s^2)^2(1+s)} = \frac{s}{(1+s^2)(1+s)^2} - \frac{s e^{-\frac{\pi}{2}s}}{(1+s^2)(1+s)^2} \quad (\text{نمره ۲})$$

$$\mathcal{L}^{-1} \left[\frac{s}{(1+s^2)(1+s)^2} \right] :$$

$$\frac{s}{(1+s^2)(1+s)^2} = \frac{(As+B)}{1+s^2} + \frac{(Cs+D)}{(1+s)^2}$$

$$\Rightarrow (As+B)(1+s)^2 + (Cs+D)(1+s^2) = s \Rightarrow \begin{cases} A=C=0 \\ B=\frac{1}{2} \\ D=-\frac{1}{2} \end{cases} \quad (\text{نمره ۲})$$

$$\Rightarrow \frac{s}{(1+s^2)(1+s)^2} = \frac{\frac{1}{2}}{1+s^2} - \frac{\frac{1}{2}}{(1+s)^2}$$