توزیع‌های رایج

توزیع‌های رایج گسسته	
Bernouli $0 < p < 1$	$p(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$ $\mu = p, \quad \sigma^2 = p(1-p)$ $m(t) = [(1-p) + pe^t], \quad -\infty < t < \infty$
Binomial $0 < p < 1$ $n = 1, 2, \ldots$	$p(x) = \binom{n}{x} p^x(1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$ $\mu = np, \quad \sigma^2 = np(1-p)$ $m(t) = [(1-p) + pe^t]^n, \quad -\infty < t < \infty$
Geometric $0 < p < 1$	$p(x) = p(1-p)^x, \quad x = 0, 1, 2, \ldots$ $\mu = \frac{p}{q}, \quad \sigma^2 = \frac{1-p}{p^2}$ $m(t) = p[1 - (1-p)e^t]^{-1}, \quad t < -\log(1-p)$
Hypergeometric (N, D, n) $n = 1, 2, \ldots, \min\{N, D\}$	$p(x) = \frac{\binom{N-D}{n-x}\binom{D}{x}}{\binom{N}{n}}, \quad x = 0, 1, 2, \ldots, n$ $\mu = n\frac{D}{N}, \quad \sigma^2 = n\frac{D}{N}\frac{N-D}{N}\frac{N-n}{N-1}$ The above pmf is the probability of obtaining x Ds in a sample of size n, without replacement.
Negative Binomial $0 < p < 1$ $r = 1, 2, \ldots$	$p(x) = \binom{x+r-1}{r-1} p^r(1-p)^x, \quad x = 0, 1, 2, \ldots$ $\mu = \frac{rp}{q}, \quad \sigma^2 = \frac{r(1-p)}{p^2}$ $m(t) = p^r[1 - (1-p)e^t]^{-r}, \quad t < -\log(1-p)$
Poisson $m > 0$	$p(x) = e^{-m}\frac{m^x}{x!}, \quad x = 0, 1, 2, \ldots$ $\mu = m, \quad \sigma^2 = m$ $m(t) = \exp\{m(e^t - 1)\}, \quad -\infty < t < \infty$

توزیع‌های رایج پیوسته

beta

$\alpha > 0$

$\beta > 0$

$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1$

$\mu = \frac{\alpha}{\alpha+\beta}, \quad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$

$m(t) = 1 + \sum_{i=1}^{\infty} \left(\prod_{j=0}^{k-1} \frac{\alpha+j}{\alpha+\beta+j} \right) \frac{t^i}{i!}, \quad -\infty < t < \infty$

Cauchy

$f(x) = \frac{1}{\pi} \frac{1}{x^2+1}, \quad -\infty < x < \infty$

Neither the mean nor the variance exists.

The mgf does not exist.

Chi-squared, $\chi^2(r)$

$r > 0$

$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{(r/2)-1} e^{-x/2}, \quad x > 0$

$\mu = r, \quad \sigma^2 = 2r$

$m(t) = (1-2t)^{-r/2}, \quad t < \frac{1}{2}$

$\chi^2(r) \Leftrightarrow \Gamma(r/2, 2)$

r is called the degrees of freedom.

Exponential

$\lambda > 0$

$f(x) = \lambda e^{-\lambda x}, \quad x > 0$

$\mu = \frac{1}{\lambda}, \quad \sigma^2 = \frac{1}{\lambda^2}$

$m(t) = [1 - (t/\lambda)]^{-1}, \quad t < \lambda$

Exponential$(\lambda) \Leftrightarrow \Gamma(1, 1/\lambda)$

F, $F(r_1, r_2)$

$r_1 > 0$

$r_2 > 0 > 0$

$f(x) = \frac{\Gamma[(r_1+r_2)/2](r_1/r_2)^{r_1/2}}{\Gamma(r_1/2)\Gamma(r_2/2)} \frac{(x)^{r_1/2-1}}{(1+r_1 x/r_2)^{(r_1+r_2)/2}}, \quad x > 0$

If $r_2 > 2$, $\mu = \frac{r_2}{r_2-2}$. If $r > 4$, $\sigma^2 = 2\left(\frac{r_2}{r_2-2}\right)^2 \frac{r_1+r_2-2}{r_1(r_2-4)}$.

The mgf does not exist.

r_1 is called the numerator degrees of freedom.

r_2 is called the denominator degrees of freedom.

Gamma, $\Gamma(\alpha, \beta)$

$\alpha > 0$

$\beta > 0$

$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad x > 0$

$\mu = \alpha\beta, \quad \sigma^2 = \alpha\beta^2$

$m(t) = (1-\beta t)^{-\alpha}, \quad t < \frac{1}{\beta}$

Laplace

$-\infty < \theta < \infty$

$f(x) = \frac{1}{2} e^{-|x-\theta|}, \quad -\infty < x < \infty$

$\mu = \theta, \quad \sigma^2 = 2$

$m(t) = e^{t\theta} \frac{1}{1-t^2}, \quad -1 < t < 1$

Logistic

$-\infty < \theta < \infty$

$f(x) = \frac{\exp\{-(x-\theta)\}}{(1+\exp\{-(x-\theta)\})^2}, \quad -\infty < x < \infty$

$\mu = \theta, \quad \sigma^2 = \frac{\pi^2}{3}$

$m(t) = e^{t\theta}\Gamma(1-t)\Gamma(1+t), \quad -1 < t < 1$

Normal, $N(\mu, \sigma^2)$

$-\infty < \mu < \infty$

$\sigma > 0$

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, \quad -\infty < x < \infty$

$\mu = \mu, \quad \sigma^2 = \sigma^2$

$m(t) = \exp\{\mu t + (1/2)\sigma^2 t^2\}, \quad -\infty < t < \infty$

t, $t(r)$

$r > 0$

$f(x) = \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\Gamma(r/2)} \frac{1}{(1+x^2/r)^{(r+1)/2}}, \quad -\infty < x < \infty$

If $r > 1$, $\mu = 0$. If $r > 2$, $\sigma^2 = \frac{r}{r-2}$.

The mgf does not exist.

The parameter r is called the degrees of freedom.

Uniform

$-\infty < a < b < \infty$

$f(x) = \frac{1}{b-a}, \quad a < x < b$

$\mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$

$m(t) = \frac{e^{bt} - e^{at}}{(b-a)t}, \quad -\infty < t < \infty$