

توزيعهای رایج

توزیعهای رایج گسسته

Bernouli

$$0 < p < 1$$

$$p(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$$

$$\mu = p, \quad \sigma^2 = p(1-p)$$

$$m(t) = [(1-p) + pe^t], \quad -\infty < t < \infty$$

Binomial

$$0 < p < 1$$

$$n = 1, 2, \dots$$

$$p(x) = \binom{n}{x} p^x(1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$$\mu = np, \quad \sigma^2 = np(1-p)$$

$$m(t) = [(1-p) + pe^t]^n, \quad -\infty < t < \infty$$

Geometric

$$0 < p < 1$$

$$p(x) = p(1-p)^x, \quad x = 0, 1, 2, \dots$$

$$\mu = \frac{p}{q}, \quad \sigma^2 = \frac{1-p}{p^2}$$

$$m(t) = p[1 - (1-p)e^t]^{-1}, \quad t < -\log(1-p)$$

Hypergeometric (N, D, n)

$$n = 1, 2, \dots, \min\{N, D\}$$

$$p(x) = \frac{\binom{N-D}{n-x} \binom{D}{x}}{\binom{N}{n}}, \quad x = 0, 1, 2, \dots, n$$

$$\mu = n \frac{D}{N}, \quad \sigma^2 = n \frac{D}{N} \frac{N-D}{N} \frac{N-n}{N-1}$$

The above pmf is the probability of obtaining x D s in a sample of size n , without replacement.

Negative Binomial

$$0 < p < 1$$

$$r = 1, 2, \dots$$

$$p(x) = \binom{x+r-1}{r-1} p^r (1-p)^x, \quad x = 0, 1, 2, \dots$$

$$\mu = \frac{rp}{q}, \quad \sigma^2 = \frac{r(1-p)}{p^2}$$

$$m(t) = p^r [1 - (1-p)e^t]^{-r}, \quad t < -\log(1-p)$$

Poisson

$$m > 0$$

$$p(x) = e^{-m} \frac{m^x}{x!}, \quad x = 0, 1, 2, \dots$$

$$\mu = m, \quad \sigma^2 = m$$

$$m(t) = \exp\{m(e^t - 1)\}, \quad -\infty < t < \infty$$

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beta $\alpha > 0$ $\beta > 0$

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha+\beta}, \quad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$
$$m(t) = 1 + \sum_{i=1}^{\infty} \left(\prod_{j=0}^{k-1} \frac{\alpha+j}{\alpha+\beta+j} \right) \frac{t^i}{i!}, \quad -\infty < t < \infty$$

Cauchy

$$f(x) = \frac{1}{\pi} \frac{1}{x^2+1}, \quad -\infty < x < \infty$$

Neither the mean nor the variance exists.

The mgf does not exist.

Chi-squared, $\chi^2(r)$ $r > 0$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{(r/2)-1} e^{-x/2}, \quad x > 0$$

$$\mu = r, \quad \sigma^2 = 2r$$

$$m(t) = (1-2t)^{-r/2}, \quad t < \frac{1}{2}$$

$$\chi^2(r) \Leftrightarrow \Gamma(r/2, 2)$$

r is called the degrees of freedom.

Exponential $\lambda > 0$

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0$$

$$\mu = \frac{1}{\lambda}, \quad \sigma^2 = \frac{1}{\lambda^2}$$

$$m(t) = [1 - (t/\lambda)]^{-1}, \quad t < \lambda$$

$$\text{Exponential}(\lambda) \Leftrightarrow \Gamma(1, 1/\lambda)$$

F, $F(r_1, r_2)$ $r_1 > 0$ $r_2 > 0 > 0$

$$f(x) = \frac{\Gamma[(r_1+r_2)/2](r_1/r_2)^{r_1/2}}{\Gamma(r_1/2)\Gamma(r_2/2)} \frac{(x)^{r_1/2-1}}{(1+r_1x/r_2)^{(r_1+r_2)/2}}, \quad x > 0$$

$$\text{If } r_2 > 2, \mu = \frac{r_2}{r_2-2}. \quad \text{If } r > 4, \sigma^2 = 2 \left(\frac{r_2}{r_2-2} \right)^2 \frac{r_1+r_2-2}{r_1(r_2-4)}.$$

The mgf does not exist.

r_1 is called the numerator degrees of freedom.

r_2 is called the denominator degrees of freedom.

Gamma, $\Gamma(\alpha, \beta)$ $\alpha > 0$

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad x > 0$$

 $\beta > 0$

$$\mu = \alpha\beta, \quad \sigma^2 = \alpha\beta^2$$

$$m(t) = (1-\beta t)^{-\alpha}, \quad t < \frac{1}{\beta}$$

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Laplace

$$\begin{aligned} -\infty < \theta < \infty \\ f(x) &= \frac{1}{2} e^{-|x-\theta|}, \quad -\infty < x < \infty \\ \mu &= \theta, \quad \sigma^2 = 2 \\ m(t) &= e^{t\theta} \frac{1}{1-t^2}, \quad -1 < t < 1 \end{aligned}$$

Logistic

$$\begin{aligned} -\infty < \theta < \infty \\ f(x) &= \frac{\exp\{-(x-\theta)\}}{(1+\exp\{-(x-\theta)\})^2}, \quad -\infty < x < \infty \\ \mu &= \theta, \quad \sigma^2 = \frac{\pi^2}{3} \\ m(t) &= e^{t\theta} \Gamma(1-t) \Gamma(1+t), \quad -1 < t < 1 \end{aligned}$$

Normal, $N(\mu, \sigma^2)$

$$\begin{aligned} -\infty < \mu < \infty \\ f(x) &= \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, \quad -\infty < x < \infty \\ \sigma > 0 \\ \mu &= \mu, \quad \sigma^2 = \sigma^2 \\ m(t) &= \exp\{\mu t + (1/2)\sigma^2 t^2\}, \quad -\infty < t < \infty \end{aligned}$$

$t, t(r)$

$$\begin{aligned} r > 0 \\ f(x) &= \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r} \Gamma(r/2)} \frac{1}{(1+x^2/r)^{(r+1)/2}}, \quad -\infty < x < \infty \\ \text{If } r > 1, \mu &= 0. \quad \text{If } r > 2, \sigma^2 = \frac{r}{r-2}. \end{aligned}$$

The mgf does not exist.

The parameter r is called the degrees of freedom.

Uniform

$$\begin{aligned} -\infty < a < b < \infty \\ f(x) &= \frac{1}{b-a}, \quad a < x < b \\ \mu &= \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12} \\ m(t) &= \frac{e^{bt}-e^{at}}{(b-a)t}, \quad -\infty < t < \infty \end{aligned}$$
