Henri Poincaré Impatient Genius

Based on a book by Ferdinand Verhulst

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- History of Dynamical Systems
- **Henri Poincaré (29 April 1854-17 July 1912)**, Vladimir Arnold(12 June 1937-3 June 2010), Stephen Smale (born July 15, 1930), ...

Different texts about Henri Poincaré

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- Papers on Topology: Analysis Situs and Its Five Supplements by Henri Poincaré, translated, with an introduction, by John Stillwell. American Mathematical Society. 2010. ISBN 978-0-8218-5234-7.–Satzer, William J. (26 April 2011). "Review of Papers on Topology: Analysis Situs and Its Five Supplements by Henri Poincaré, translated and edited by John Stillwell". MAA Reviews, Mathematical Association of America. Archived from the original on 26 January 2024. Retrieved 26 January 2024.
- Verhulst, Ferdinand, 2012 Henri Poincaré. Impatient Genius. N.Y.: Springer.
- Henri Poincaré, l'œuvre scientifique, l'œuvre philosophique, by Vito Volterra, Jacques Hadamard, Paul Langevin and Pierre Boutroux, Felix Alcan, 1914.

Verhulst, Ferdinand, 2012 Henri Poincaré. Impatient Genius. N.Y.: Springer.

One sunny afternoon, on the outskirts of Nancy, in the French province of Lorraine, a woman was walking with her children, Henri and Aline. They were walking along a brook with paths on both sides, connected by several bridges. Little Henri, who was two years older than his sister, often ran on ahead with his dog, Tom. Suddenly, he noticed that his mother and sister had crossed the river and were walking on the other side. Henri's mother gestured to him that he could cross at the next bridge, but he immediately jumped into the water, which came up to his waist, and dashed across the river to rejoin his mother and sister.

This direct style of solving problems was typical of Henri Poincare for the rest of his life. Also the impatience.

- Part I The Life of Henri Poincaré
- Part II Scientific Details and Document

Part I The Life of Henri Poincaré

- The Early Years
- Academic Education: 1873-1879
- Impressive Results in Vesoul and Caen
- Career in Paris
- The Prize Competition of Oscar II
- Philosophy and Essays
- At the End, What Kind of a Man?

Aside from his exceptional intelligence, Henri also had an exceptional memory. His dissertation advisor and later colleague Gaston Darboux observed that it was probably not very well known how much Henri Poincaré knew Â´ when he was young. He had only to read a book once to know its contents in their entirety; he could recall on what page and on what line of that page a specific item in the book could be found. For years following a trip abroad, he could recite all the stations at which the train had stopped and in addition, the names of all the towns and hotels where they had stayed. With such a memory, it is no surprise that he was able to master his lessons at school without making any notes.

During the period 1867-1868, when Henri was nearly fourteen, one of his teachers at the secondary school reported to his mother, Eugenie, “Henri will be a mathematician.” When she appeared not to understand, he added, “I mean a great mathematician.” However, Henri himself told Aline at the time, “I cannot commit myself to anything. I donât know what I will do in twenty yearsâ time.” This seems a natural attitude for a boy of that age.

In October 1874, his first research paper appeared, and of course its topic was geometry: “Demonstration nouvelle des propri et es de l’indicatrice dâune surface” appeared in Annales de Math ematiques, 2e serie, vol. XIII. It is an interesting exercise in curvature and osculating surfaces. It certainly should have made an impression that a student had written such a paper on his own and had it published, but what did the professor of geometry think about it? That will become clear indirectly.

In June 1878, Henri's studies at the Ecole des Mines came to an end. The final ranking had Henri third, after Bonnefoy and Petitdidier, but the result did not seem to interest him. The following year, in March 1879, Henri Poincaré, 24 years old, was appointed to the post of mining engineer in Vesoul, relatively close to Nancy. The appointment was made by the National Inspection of Mines, with the formulation "appointment to ordinary mining engineer of the third class charged with the mineralogical subdistrict Vesoul and in addition the supervision of the railways in the east." As we will see, the job was not without danger. Bonnefoy and Petitdidier also became mining engineers. They died in their late twenties from accidents that occurred in the course of their duties. Henri's activities in Vesoul were short-lived. In December 1879, he was appointed to a lectureship of mathematics at the Faculte des Sciences of Caen, in Normandy. Formally, he remained his whole life a member of the corps of mining engineers. On June 16, 1910, he was appointed inspector-general of mines, in this case most likely an honorary title.

During all the activities of these years, Henri's mathematical discussions and research had never been interrupted. It seems that Henri almost casually wrote his dissertation during his second and third years at the Ecole des Mines; for mathematical details, see Section 9.1. Its inspiration was from a paper by Briot and Bouquet in the Journal de l'Ecole Polytechniqe dealing with solutions of differential equations. As a first result, Henri wrote a short paper, which he submitted to that same journal. The dissertation, titled Les propri et es des functions d efinies par des equations aux d eriv ees partielles, was submitted at the turn of the year 1877-1878 to his supervisors Darboux, Laguerre, and Bonnet.

In Vesoul, Henri divided his time between his work as a mining engineer and mathematics. He was qualified in both, and he practiced both professions. In that period, his cousin Raymond Poincaré wrote two novels. In Vesoul, Henri also wrote a novel, a romantic story that never was published.

On December 1, 1879, Henri Poincaré was appointed as a lecturer in mathematical analysis at the University of Caen.

Poincaré was appointed to the position of ma Ã® tre de conferences in the Faculte des Sciences at the Sorbonne in Paris on October 19, 1881. This type of position had only recently been instituted; the idea was to attach to the holder of a regular chair a sort of tutor or coach, a qualified lecturer who looked after his students, gave them problems and exercises, and corrected their papers. Apart from those tasks, this ma Ã® tre could follow his own independent line of research. Until his death in 1912, 31 years later, Poincaré continued to live and work in Paris, exhibiting extraordinary creativity and productivity in many fields.

Even when he was very young, Henri Poincaré was interested in what happened in his town, in his country, and in the world.

A number of Henri Poincaré's essays are usually classified as "philosophical." Most of them have been collected in six books.

The latter turned out to be a mathematical prodigy, a genius who was always far ahead of everybody else working on the same topics. While Klein and his students were working on special problems, Poincaré formulated the theory from the outset in great generality.

How did he become so productive? What made him a genius? Apart from his emotionally sound and balanced spirit, we can note three factors: his high intelligence, an exceptional memory, and the total and permanent obsession with scientific problems. He truly never stopped thinking about science.

The analysis was carried out at the Psychological Laboratory in Paris, resulting in a number of not very exciting observations. For instance:

- He worked during the same times each day for short periods. Mathematical research took four hours a day, two in the morning and two hours from 5 p.m. till 7 p.m.
- His normal work habit was to solve a problem mentally and then write it down.
- He was ambidextrous and near-sighted.
- He could very well memorize and visualize what he read and heard.
- He was physically clumsy and not artistically gifted.
- He was always in a hurry and hated going back for corrections.
- He believed in letting his unconscious work on a problem while he consciously worked on another problem.

Most remarkable about Henri Poincaré are his versatility and his creation of whole new research fields, including the following:

- Automorphic functions, uniformization
- The qualitative theory of differential equations
- Bifurcation theory
- Asymptotic expansions, normal forms
- Dynamical systems, integrability
- Mathematical physics
- Topology (analysis situs)

Scientists are often in competition, constantly evaluating their colleagues and comparing themselves to them. Richard Feynman wittily called one scientist a “big shot,” another one a “small shot.” How should Poincaré’s stature be evaluated in comparison with scientists of the last 150 years? One name comes immediately to mind: David Hilbert, an eminent mathematician, although more restricted in his choice of research topics. The styles of both men were, however, very different, and trying to compare Hilbert and Poincaré makes little sense. Henri Poincaré was in a class by himself.

Part II Scientific Details and Documents

- Automorphic Functions
- Differential Equations and Dynamical Systems
- Analysis Situs
- Mathematical Physics
- Poincaré's Address to the Society for Moral Education
- Historical Data and Biographical Details

Differential Equations and Dynamical Systems

Poincaré's Thesis of 1879:
Henri Poincaré presented his thesis to the Faculté des Sciences of the University of Paris to obtain the degree of doctor of mathematical sciences. The title: "Sur les proprietéstés fonctions définies par les équations aux différences partielles." It was accepted on August 1, 1879, by a committee consisting of J.-C. Bouquet (chairman), P.-O. Bonnet, and G. Darboux.

A Revolutionary Memoir on Differential Equations, 1881-1882: Poincaré-Bendixson theory

Differential Equations and Dynamical Systems

The three volumes of the Mécanique Céleste [Henri Poincaré. Les Méthodes Nouvelles de la Mécanique Céleste, 3 vols. Paris: Gauthier-Villars, 1892, 1893, 1899.] together form the first modern textbook on dynamical systems (33 chapters).
Poincaré's key results in dynamical systems can be listed as follows:

- Poincaré expansion with respect to a small parameter around a particular solution of a differential equation (Chapter 2).
- The Poincaré-Lindstedt expansion method (Chapter 3) as a continuation method and as a bifurcation method for periodic solutions.
- Characteristic exponents and expansion of exponents in the presence of a small parameter; exponents when first integrals exist (Chapter 4).
- The famous proof that in general, for time-independent Hamiltonian systems, no other first integrals exist besides the energy (Chapter 5).
- The idea of "asymptotic series" as opposed to convergent series (Chapters 7 and 8).

- Chapter 1: Generalities and the Method of Jacobi
- Chapter 2: Series Expansions
- Chapter 3: Periodic Solutions
- Chapter 4: Characteristic Exponents
- Chapter 5: Nonexistence of Uniform Integrals
- Chapter 6: Approximation of the Perturbation Function
- Chapter 7: Asymptotic Solutions
- Chapter 8: Formal Aspects of Convergence
- Chapter 9: The Methods of Newcomb and Lindstedt
- Chapter 10: Secular Variations
- Chapter 11: Application to the Three-Body Problem
- Chapter 12: Application to Orbital Calculations
- Chapter 13: The Divergence of the Lindstedt Series
- Chapters 14-15: The Direct Calculation of the Series
- Chapter 16: The Methods of Gyldén
- Chapter 17: The Case of Linear Equations
- Chapter 18: The Case of Nonlinear Equations
- Chapters 19-20: The Methods of Bohlin
- Chapter 21: Extension of the Method of Bohlin
- Chapter 22: Integral Invariants
- Chapter 23: The Formulation of Integral Invariants
- Chapter 24: The Use of Integral Invariants
- Chapter 25: Integral Invariants and Asymptotic Solutions
- Chapter 26: Stability in the Sense of Poisson
- Chapter 27: The Theory of Consequents (Poincaré Map)
- Chapter 28: Periodic Solutions of the Second Kind (Superharmonics)
- Chapter 29: Forms of the Principle of Minimal Action
- Chapter 30: The Formation of Solutions of the Second Kind
- Chapter 31: Properties of Solutions of the Second Kind
- Chapter 32: Periodic Solutions of the Second Kind
- Chapter 33: Doubly Asymptotic Solutions

The Prize Competition of Oscar II:
three-body problem: chaotic dynamics, planar restricted three-body problem

Apart from the fundamental celestial mechanics books, he has also lecture notes. Here we list the topics of the lecture notes:

- Celestial mechanics (the Leçons)
- The mathematical theory of light, two volumes
- Electricity and optics, two volumes (Maxwell theory and Hertz oscillations, wireless telegraphy)
- Thermodynamics
- The theory of elasticity
- Vortical motion ("tourbillons")
- Electrical oscillations
- Capillarity
- Analytical theory of the propagation of heat
- Probability
- Potential theory
- Kinematics and fluid mechanics
- Equilibrium figures of a fluid mass
- Cosmogonic hypotheses

Notice: The texts in slides 3 and 4 are from *https://en.wikipedia.org/wiki/Henri-Poincaré* and the texts in slides 6-30 are from "Verhulst, Ferdinand, 2012 Henri Poincaré. Impatient Genius. N.Y.: Springer." and other related references that this book refers them i.e.

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- Aline Boutroux. Vingt ans de ma vie, simple vérité. Nancy: Archives B Centre d'Études et de Recherches Henri Poincaré, 1912.
- C. Briot and T. Bouquet. "Recherches sur les propriet es des fonctions definies par les equations diff erentielles." J. de l'Ecole Polytechnique, Cahier 21 (1856), 133-198.
- Gaston Darboux. "Eloge historique d'Henri Poincaré, 1913."

"Continued on the next slide"

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- Henri Poincaré. Les Méthodes Nouvelles de la Mécanique Céleste, 3 vols. Paris: Gauthier-Villars, 1892, 1893, 1899.
- Henri Poincaré. "Note sur les propriétés des fonctions d efinies par les équations différentielles." J. de l'École Polytechnique, Cahier 45 (1878), 13-26.
- Henri Poincaré. Lecture notes.
- E. Toulouse. Henri Poincaré. Paris: Flammarion, 1910.

Thank You

Question?